A Simplified Multiattribute Procurement Auction with Postponed Scoring by a Double Revelation Mechanism

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Abstract

This paper provides a multiattribute procurement auction that eases the principal's articulation of preferences. The suggested procurement auction applies yardstick techniques to enhance the multi-dimensional competition and to postpone the Principal's scoring (the weighting of attributes). The price-dimension in the submitted bids are replaced by yardstick prices defined by a linear envelopment of the other bidders' bids when applicable. The resulting yardstick bids are revealed to the Principal, who selects the most preferred among these. The Principal's choice reveal a potential cone of linear scoring functions, consistent with the principal's choice given linear preferences. The resulting scoring functions are used to score the original bids. While the highest score wins the second highest score settle the compensation. If the auction results in more than one highest scoring bid, the Principal ends the auction by selecting the most preferred.

The auction provides almost ideal incentives for bidders to reveal prices less than or equal to true cost. In general bidding below true cost involves a risk of getting a loss, nevertheless it may also both increase the chance of winning and result in a higher compensation if winning. This strengthens the Principal's articulation of preferences and the competition in general. Also, the postponed scoring lowers the Principal's transaction costs involved in tender description and thereby making the suggested auction a closer alternative to a traditional negotiation while ensuring transparent and strong competition.

Keywords: Auctions, multi-attribute, yardstick competition, market design.

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1 Introduction

Efficient procurement is crucial for the success of any public and private organization. While minimizing the spending is important other attributes such as various types of qualities, delivery time etc. may be equally important. Consequently, the development of procurement systems faces an ongoing challenge in designing trading systems that facilitate transparent competition on both price and multiple attributes as well as ensuring sufficient flexibility for operational purposes. The main challenge is the balancing of the seller's costs of delivering different attributes and the buyer's benefits of these. This is unlike a traditional price-only auction, where the Principals preferences are well known.

In a multiattribute auction the asymmetry of information is two-sided. On one side, the Agents' true costs of delivering a given bundle of attributes is private information to each agent and on the other side, the Principal's willingness to pay for a given bundle of attribute is private information to the Principal. While a traditional negotiation process allows full flexibility in this two-sided matching, it is typically ill-structured and opaque. On the contrary, traditional multi-attribute auctions specify transparent rules for how this balancing of costs and benefits takes place. This however, is a less flexible solution and requires that the Principal defines his preferences *a priori* and sometimes with very limited information about what the sellers have to offer.

This paper provides a multiattribute auction that simplifies the balancing of costs and benefits without limiting the competition. The suggested auction imitates the flexibility of a negotiation by allowing the Principal (buyer) to review competitive pre-processed bids, which eases the Principal's articulation of preferences. The pre-processed bids and the Principal's choice, are used to estimate one or more linear scoring functions that are consistent with the Principal's choice. The properties of the auction is driven by two different embedded revelation mechanisms: 1) the price-bids are replaced by yardstick prices defined by a linear envelopment of the other bidders' bids when applicable and 2) the endogenous settled linear scoring functions are used in an embedded second scoring mechanism. The suggested auction provides almost ideal incentives for the bidders to reveal prices no higher than their true cost. However, it may be optimal for a bidder to bid slightly below true cost in particular if the agents expect little competition, which in turn enhance the competition in the most critical situation of a thin market. Hereby, the suggested auction enhance the competition as well as strengthen the Principal's articulation of preferences.

The outline of the paper is as follows. Section 3 defines the context and introduce more backgrounds on related multiattribute auctions. Sections 4 describes the suggested multiattribute auction and its properties and 5 discuss extensions. Section 6 concludes.

2 Relation to existing literature

There are several practical instances of multi-attribute auctions, e.g. the conservation reserve program in the USA (Babcock et al. (1997), Vukina et al. (2008)), the Department of Defence procurement auctions for weapon systems in the USA (Che, 1993) and the television franchising in the UK (Galapo, 1999). Also, some of the systems used for e-procurement have several similarities with multi-attribute auctions, for example by the way they automate negotiations (see e.g. Burmeister et al. (2002) for an introduction to some of these systems). As an example of this, there are several papers suggesting an incorporation of multi-dimensional auctions into the so-called Request for Quote (RFQ) systems, see e.g. Milgrom (2000) and Bichler et al. (2003). RFQ systems uses the

Internet to improve the searching and matching process between buyers and sellers in general.

Though, unlike auctions in general, the theoretical literature on multi-attribute auctions is relatively sparse. The most well-known multi-attribute auction is the *score auction.* The score auction uses a score function to map multi-dimensional bids into a one-dimensional score. The score reflects the principal's utility or the welfare function and is used to allocate an item or task and to determine the price. In a traditional score auction the scoring function is made public before bidding, it is therefore assumed that the principal have well articulated preferences *a priori.*

Thiel (1988) uses tools from consumer theory to show that the optimal multi-attribute auctions will be equivalent to the design of one-dimensional auctions. He considers the situation where the procurer decides on a budget, which becomes known to the agents. Also, the procurer does not value any savings. It is unclear whether the assumptions of the preset budget and, especially, valueless savings are appropriate in most procurement situations according to Branco (1997). The paper sketches the idea of the multi-dimensional score auction, but it offers no proof of optimality for such an auction.

Che (1993) is a central paper on multiattribute score auctions. He shows how the existing theory on auctions can be generalized to multiattribute auctions. Using the *revenue equivalence theorem* as it is presented by Riley and Samuelson (1981), he proves that the *first score* and the *second score auction* leave the principal with the same utility. In a first score auction, the bidder with the highest score wins and has to meet the highest score. In a second score auction, the bidder with the highest score wins and has to meet the second highest score. Che arrives at his strong result by restricting the bidders' cost type to be monotonic. The cost advantages and disadvantages are universal such that types can effectively be given a one-dimensional ordering.

Branco (1997) shows that Che's model fails with correlated costs among the agents. In such cases, the optimal quality will depend on all bidders' cost of producing the quality. To solve this problem, he introduces a 2 stage model. The first stage selects a winner and the second stage determines an optimal quality based on all bids submitted. He argues that this is typically what happens. The US defense auctions, for example, typically have a 2 stage system where the quality is negotiated after the winner has been found.

A more recent paper is Parkes and Kalagnanam (2005) that provides an iterative multi-attribute auction in which the Principal provides an initial valuation function, and the Agents compete in an open auction where the auctioneer evaluate each bid by computing the bid that maximizes the Principal's preferences. It is shown that the auction terminates at the Vickrey Clarke Growes (VCG) allocation after a series of bidding rounds. The Principal states his preferences *a priori* and the auction approximate a VCG allocation after a potentially large number of rounds. The suggested auction in this paper differs in various ways, e.g. it requires no a priori valuation function, it consist of a single bidding round only and it applies a different winner/price selection method.

Another recent paper Bogetoft and Nielsen (2008) provides a multiattribute auction that combines yardstick competition and the traditional second score auction. The use of yardstick competition introduces pricing relative to the other bidders' bids as oppose to a second score auction where the score is used for both selecting the winner and pricing the good. Hereby, a priori information about the overall cost structure can be utilized by the Principal to extract information rent. While the suggested auction in this paper rely on the same pricing principles it differs in various ways, e.g. it requires no a priori scoring function and it applies a different winner/price selection method.

In practice, there are no particular reasons to expect that the score function is commonly known a priori by the Principal. The determination of $V(.)$ may be complicated either for intraor interpersonal reasons. When the principal is a single person and $V(.)$ represents the principal's intra-personal trade-offs. The complication of this is represented by a large literature on Multiple Criteria Decision Making (MCDM) treating this situation, see e.g. Bogetoft and Pruzan (1997). When the Principal represents a number of persons, the construction of $V(.)$ may involves interpersonal conflicts. The complication of this is no smaller as reflected by the literature on Social Choice, e.g. Arrow (1963) and Mas-Colell et al. (1995).

Practice support the assumption that determination of such a score is a difficult matter. In the conservation reserve program the USDA (United States Department of Agriculture) rank the bids into a score. The actual determination of these scoring rules has been widely discussed, see e.g. Babcock et al. (1996, 1997). The applied scoring has also been an issue in the wholesale market for electricity in California. Here the choice of an unsuitable scoring rule had severe consequences, see Bushnell and Oren (1994) and Chao (2002).

Nevertheless, the EU procurement directives requires an explicit weighting of the different attributes a priori, for trades that exceed a certain threshold value. However, the directive does allow the Principal to *"derogate from indicating the weighting of the criteria (...) where the weighting cannot be established in advance, in particular on account of the complexity of the contract. In such cases, they must indicate the descending order of importance of the criteria."* For more see Union (2004).

There are only a few papers relaxing the assumption of a priori given value function in the literature on multi-attribute auctions. One contribution is Cripps and Ireland (1994) they investigates the issues of setting quality thresholds that are unknown to the bidders. In the present set-up, this corresponds to the use of simple value functions with $V(y) = 1$ if $y \geq t$ and 0 otherwise, where $t \in \mathbf{R}_0^s$ is the quality threshold. Another is Beil and Wein (2003) who studies the sequential learning of the welfare function and the bidders' cost functions by a sequence of score auctions with different score functions. However, Beil and Wein (2003) does not directly address the risk of strategic bidding and basically presumes truthful revelation in the sequence of trial auctions.

Bogetoft and Nielsen (2008) discuss the problems with strategic bidding in case of uncertain scoring related to the suggested hybrid auction described above. Let us assume that the Principal can commit to select according to his values, but not to paying according to his true values. In general we have a finite set of score functions $\tilde{\mathcal{V}} \in \mathcal{V}$ that select the winner (truthfully selected). To control the possible opportunistic behavior the Principal have to be restricted in the choice of $V \in \tilde{\mathcal{V}}$. However, even if the Principal commit to select the most profitable scoring function from the winning bidder's point of view, strategic bidding is not avoided.

Consider the hybrid auction in Bogetoft and Nielsen (2008) and a situation where the Principal commits to a set of feasible scoring functions and to mitigate manipulation from the bidders' point of view, we propose to pay the winner the maximal amount that is consistent with choosing him as a winner. However, this does not make truth-telling a dominating strategy as illustrated in Figure 1. A priori the Principal has committed to select one of two different scoring functions. Assume that the bidders 1, 2 and 3 have submitted a single bid each representing their true cost. Upon receiving the bids the Principal prefer V^1 and bidder 2 is selected. Now, ex post bidder 1 could have benefitted by manipulating his cost bid such that the bid is just selected by V^1. Hereby bidder 1 would have won and the compensation determined by the yardstick cost $b^{\hat{1}}$, i.e. the maximal amount that is consistent with choosing bidder 1 as a winner. In general, since the selection criteria is the most important this mechanism makes it optimal to bid below true cost to maximize expected utility. However, bidding below true cost involve a chance of under compensation.

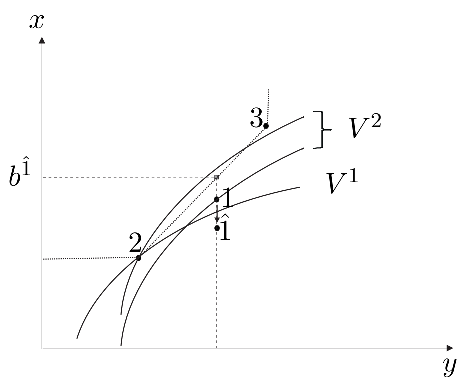

Figure 1: Finite scoring functions

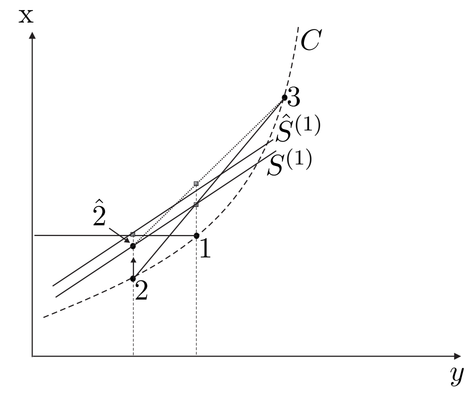

Figure 2: Infinite linear scoring functions

Now, consider the special case where the set of score functions $\tilde{\mathcal{V}} \in \mathcal{V}$ is infinite and restricted to linear scoring functions–a case that relates some how to the suggest auction in Section 4. If the Principal commits to the same principal as before and pay the winner the maximal amount that is consistent with choosing him as a winner, the auction would be equivalent to base scoring entirely on the yardstick cost presented in Section 3.2, i.e. using $S(x^i, y^i) = V(y^i) - C^{-i}(y^i)$. To see this not that $C^{-i}(y^i)$ is constructed by the closest linear envelopment of the other bidder's bids.

Unfortunately, such an auction is not incentive compatible either. To see this, consider bidder i with the highest reservation score. Now, a "neighbor bidder" to i (effectively helping to span the yardstick price of i), may influence the winner's chance of winning. Hereby the "neighbor bidder" may also affect his own chances of winning the auction by putting the otherwise winning bid in a less favorable position, as illustrated in Figure 2. Bidder 2 is the "neighbor bidder" to the original winner bidder 1. Now bidder 2 can win the auction by increasing his price-bid to $\hat{2}$ and hereby increase the yardstick cost for bidder 1. This type of manipulation is addressed in the suggested auction in Section 4.

3 Setting

In this section, the procurement setting and the applied yardstick prices are presented.

3.1 Procurement setting

Consider a private or public procurement of an item described by a vector of attributes in a market with n potential sellers. By fixing the different attributes at given levels, the Principal can (in most cases) design an auction that will minimize the procurement cost for these quality levels. The outcome, however, may not be Pareto efficient, i.e. that there is an optimal trade-off between the seller's costs of alternative quality levels and the buyer's benefits here from. To make these trade-offs, we need more advanced multi-dimensional forms.

Throughout the paper, consider a Principal seeking to procure a good or a project from one of n risk neutral Agents, $i \in N = \{1, ..., n\}$.

Let the properties of the good offered by agent i be defined by a s-dimensional vector of outputs $y^i \in \mathbf{R}_0^s$ and let $Y^i \subseteq \mathbf{R}_0^s$ be the set of possible output profiles for agent i. One can think of the output vector as defining a bundle of different goods or as the characteristics or qualities of a given good. To focus on the adverse selection problems of private information ex ante (as opposed to subsequent moral hazard problems) we assume that the characteristics of the product is verifiable and that delivery of the promised qualities can therefore be costlessly enforced (e.g. by a harsh penalty for deviations).

To produce the output vector y^i, agent i needs inputs. Let inputs be aggregated into a single dimensional cost $c^i \in \mathbf{R}_0$. These costs cannot be verified by a third party and are in general private information to the agent. The principal just know that $c = (c^1, \ldots, c^n)$ belongs to some (unknown) set $\mathbb{C} \subseteq \mathbf{R}_0^n$.

Since the actual output is verifiable, possible manipulations regards the costs. Therefore, the signal from agent i is simply a price-output bid

$$(x^i, y^i) \in \mathbf{R}_0^{1+s}$$

with the interpretation that agent i is willing to produce y^i if he is paid at least x^i.

We assume throughout that the aim of the agent is to maximize (expected) profit:

$$\pi^i = b^i - c^i$$

where b^i is the payment to bidder i.

The aim of the principal is to maximize (expected) net value, i.e. the value that the good generates minus the costs of producing or acquiring it. If he is a social planner seeking to maximize social value, his objective value when agent i is selected is:

$$V(y^i) - c^i$$

where $V(.)$ is a weakly increasing value function. More generally, we could let the objective be $V(y^i) - (1+\epsilon)c^i$, where $\epsilon \geq 0$ reflects the possible costs of distortions from the collection of public funds. Since it has no influence on the results–except that all costs are increased by a factor $(1+\epsilon)$–we simply assume that $\epsilon = 0$ here.

If the principal is a private, profit maximizing entity, his net value when agent i is selected, is:

$$V(y^i) - \sum_{i=1}^{n} b^i$$

i.e. his value minus his payment to the agents.

3.2 The Bidders' Cost Structure and Yardstick Prices

As mentioned above, the Principal just know that c^i, bidder i's costs of producing and delivering the good or bundle of goods described by y^i, belongs to some unknown set $\mathbb{C} \subseteq \mathbf{R}_0^n$. So basically, it is assumed that there exists some underlying common cost structure $C(y)$.

Throughout the paper the idea of a common but unknown cost structure is used as vehicle to extract information rent as well as to postpone the articulation of preferences as suggested in the Section 1. In general, such affiliated valuations may allow the Principal to further extract information rent, e.g. suggested by Milgrom and Weber (1982) and Cremer and McLean (1988).

Now, let $C(y) = \min\{x \mid (x, y) \text{ is feasible}\}$ be a cost function having the following properties:

$$\begin{array}{ll} A1. & \text{Weakly increasing} : y' \geq y \Rightarrow C(y') \geq C(y), \\ A2. & \text{Convex} : C(\gamma y + (1-\gamma)y') \leq \gamma C(y) + (1-\gamma)C(y'), \forall \gamma \in [0, 1]. \end{array}$$

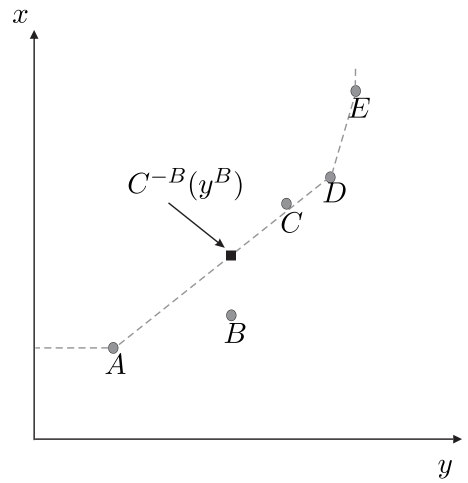

Figure 3: Convex cost structure

A number of different cost models fulfill these two simple axioms, and the suggested auction in Section 4, rely on the existence of an underlying cost structure that is convex and weakly increasing as the one shown in Figure 3.

For computational purposes, the cost structure may be modeled with linear programming based on the submitted bids as input data: $z^i = (x^i, y^i), i \in I$. As such the yardstick price $C^{-i}(y^i)$, as illustrated in Figure 3, is defined as follows:

Definition 1. *For all $i \in N$ the yardstick price $\bar{x}^i$ is computed as:*

$$\bar{x}^i = C^{-i}(y^i) = \inf\{x \in \mathbf{R}_0 \mid x \geq \sum_{j \in I, j \neq i} \lambda^j x^j, \quad y^i \leq \sum_{j \in I, j \neq i} \lambda^j y^j, \sum \lambda = 1, \lambda^j \geq 0\}.$$

The above solution identifies a single point $(\bar{x}^i, y^i)$ on the spanned frontier (being the smallest convex envelopment of all bids except for bidder i's bid).

Note that for some values of y, the associated yardstick costs may be infinite, i.e. the estimate of the technology suggests that it may be possible to produce y at any costs. In the suggested auction in Section 4 this is dealt with by asking the Principal to announce a upper bound on the bids, i.e. the most extreme value of attributes and the Principal's associated reservation price.

4 The Suggested Multiattribute Auction

In this section, we present the rules defining the auction and proof important properties of the auction.

The suggested multiattribute auction simplifies the procurement process by postponing the Principal's articulation of preferences. Therefore, the invitation for tenders only describes the attributes that is taken into consideration, not the Principal's relative weighting of price and attributes.

Initially we assume that output $y^i \in Y^i \subseteq \mathbf{R}_0$ is one-dimensional and that only a single output level can be produced for each agent, i.e., $Y^i = \{y^i\}$ for all $i \in N$. This may represent a situation where specific investments by the agents (bidders) restrict them to a single value of y. These assumptions are further discussed in Section 5.

The rules defining the auction is provided below.

The auction rules:

Step 0: The Principal announces the procurement proposal and a upper bound on the bids $z^P = (y^P, x^P)$, where y^P is the highest acceptable value of y and x^P is the highest acceptable price for y^P.

Step 1: Each participant $i \in N$ submit a single sealed bid $z^i = (x^i, y^i)$. Let Z be the set of bids including z^P, i.e., $Z = \{z^i\}_{i \in N} \cup z^P$.

Step 2a: The TTP computes a yardstick price $\bar{x}^i = C^{-i}(y^i)$ for all $i \in N$ (see definition 1) that replaces x^i when $x^i \leq \bar{x}^i$. Let $\bar{z}^i = (\bar{x}^i, y^i)$ be a yardstick bid and let $\bar{Z}^i$ the set of yardstick bids for $i \in N$.

Step 2b: An additional artificial yardstick bid is constructed as $z^0 = (\bar{x}^0, 0)$ where $\bar{x}^0 = \min_{i \in N}\{\bar{x}^i\}$. Let $\bar{Z} = \bar{Z}^i \cup \{z^P, z^0\}$.

Step 3: The set $\bar{Z}^i$ is presented to the Principal, who selects a single element $\bar{z}^{i^*}$.

Step 4: The yardstick bids in $\bar{Z}$ are used to construct k ($k = 1, 2.$) linear scoring functions, $s^i_k = a_k + b_k y^i$ with vertex in $\bar{z}^{i^*}$ (see definition 3 below).

Step 5: The TTP compute the scores $S^i_k = s^i_k - x^i$ for all $i \in N$ and $k = 1, 2$.

Step 6a: The TTP identify the highest scoring bidders with respect to each scoring function as $i^{(1)}_k = \arg\max_{i \in N}\{S^i_k\}, k = 1, 2$.

Step 6b: The TTP identify the second highest scoring bidders with respect to each scoring function as $i^{(2)}_k = \arg\max_{i \in N \setminus i^{(1)}_k}\{S^i_k\}, k = 1, 2$.

Step 7: If there exist a bidder $i' \in N$ for which $S^{i'}_1 = S^{(1)}_1$ and $S^{i'}_2 = S^{(1)}_2$ then i' is sole winner and the second highest score is determined as $S^{(2)}_1 = S^{(2)}_2 = \min\{S^{(2)}_1, S^{(2)}_2\}$. Otherwise the Principal selects the final winner in Step 8.

Step 8: The TTP computes the potential compensation $b^{i^{(1)}_k} = s^{i_k} + S^{(2)}_k$ for each of the selected highest scoring bids from Step 7, and presents the resulting offers $z^{i^{(1)}_k} = (b^{i^{(1)}_k}, y^{i^{(1)}_k})$ to the Principal, who selects the final winner i^{**} among these.

Step 9: The winner i^{**} is compensated by $b^{i^{**}}$ and losers are not compensated.

Using the following definitions:

Definition 2. *Consider the yardstick bid, $\bar{z}^{i^*}$, of the selected agent i^*. Given $\bar{z}^{i^*}$ the neighbor bid to the left ($\bar{z}^{i^l}$) and to the right ($\bar{z}^{i^r}$) are defined as:*

$\bar{z}^{i^l} = \arg\max_{z \in \bar{Z}}\{z | y^i < y^{i^*}\}$

$\bar{z}^{i^r} = \arg\min_{z \in \bar{Z}}\{z | y^i > y^{i^*}\}$

Definition 3. *Given the Principal's selection of yardstick bid $\bar{z}^{i^*}$ the basic scoring functions $s_k^i = a_k + b_k y^i$ represent an envelopment of the left and right neighbor yardstick bids with the Principal's choice as vertex (see illustration in Figure 4). The two constants a_k and b_k are given as:*

$$b_k = \frac{|\bar{x}^{i^*} - \bar{x}^k|}{|\bar{y}^{i^*} - \bar{y}^k|}$$

$$a_k = \bar{x}^{i^*} - \frac{|\bar{x}^{i^*} - \bar{x}^k|}{|\bar{y}^{i^*} - \bar{y}^k|}\bar{y}^*, \text{ for } k = i^l, i^r.$$

The construction of the scoring functions is based on the idea of *Value Efficiency* as presented in Halme et al. (1999). As such, the spanned frontier may be interpreted as boundaries on the largest set of linear preferences consistent with the Principal's choice z^{i*}.

The auction rules are further described and illustrated below by a numerical example (provided in Section 4.1).

The Principal initiates the problem by publicly announcing the single bid z^P stating the maximum value of the attribute in question y^P and the Principal's reservation price x^P for y^P. z^P enters the auction both as a submitted bid as well as a yardstick bid. Hereby, x^P addresses the problem that there does not exist a yardstick price for the most extreme value of y.

The auction is organized into two processes: A bidding process that ends by the Principal's choice in Step 3 and a scoring process that finalizes the auction.

The bidding process: In Step 1 the bidders submit sealed multiattribute bids. In Step 2a, the yardstick prices $(C^{-i}(y^i))$ are computed as illustrated in Figure 3 and defined in definition 1. If $C^{-i}(y^i) \geq x^i$ the bidders are assigned a yardstick bid by replacing the submitted price-bid with the yardstick price. Step 2b computes an artificial yardstick bid that might be used to span the scoring functions in Step 4. In Step 3, the Principal review the yardstick bids and select a single yardstick bid. Figure 4 illustrates a situation where 4 bidders have submitted a single bid each and where the applicable bidders A, B and C are assigned a yardstick bid illustrated by the squares.

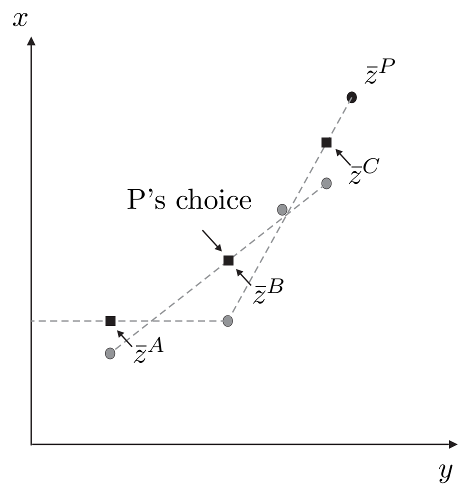

Figure 4: All yardstick bids and the Principal's choice

The scoring process: In Step 4 the Principal's choice in Step 3 is used to deduct information about the Principal's preferences. The Principal's choice provides an ordinal top priority and a focal point for the applied linear scoring functions which is estimated in Step 4 and further defined in definition 2. In Step 5 each bid is given a score by the difference in the price-dimension between the submitted price-bid and the estimated scoring functions. Figure 5 illustrates bidder B's score S_1^B with respect to scoring function 1. In Step 6a-6b the bidders' with respectively the highest and second highest scores for each of the scoring functions are identified. If a single bidder holds the highest scoring bid with respect to both scoring functions, then this winner is the sole winner (Step 7). Otherwise, the Principal selects the final winner among the highest scoring bids after knowing the compensation (Step 8). Step 9 finalize the auction by compensating the selected winner. Figure 6 illustrates the final scoring process. Here the auction identify a single winner B and the compensation is bounded by bidder A ($S_2^{(2),A}$) and given as $b^B = s_1^B + S_1^{(2),B}$.

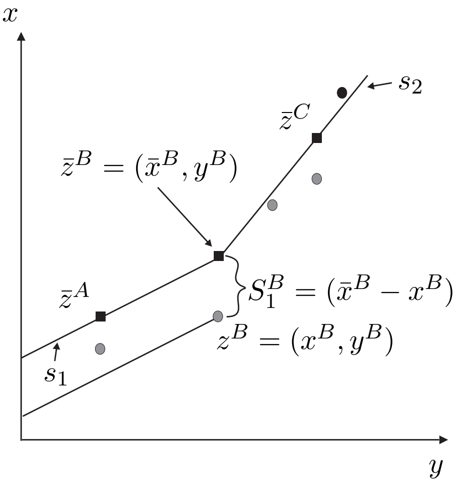

Figure 5: B's score

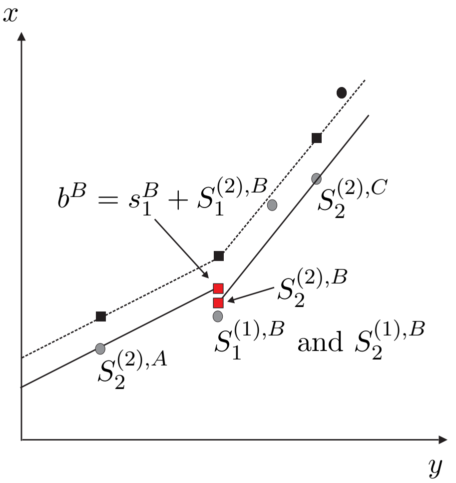

Figure 6: The final scoring

We now record the first somewhat surprising result.

Proposition 4. *If the Principal's selection of preferred yardstick bid ($\bar{z}^{i*}$) is unchanged by agent i's increased cost bid, i has no incentive to bid above true cost.*

Proof. Consider an arbitrary agent $i \in N$. First, note that i's bid has no influence on i's computed yardstick bid $\bar{z}^i$. Second, if i bids above his true cost, $x'^i > x^i$, then both neighbor yardstick bids of i increases: Indeed, i has a neighbor yardstick bid to the left $\bar{z}^{i^l}$ and to the right $\bar{z}^{i^r}$ (these exists since z^P and z^0 are added). The yardstick bid, $\bar{z}^{i^l}$, is determined by z^i and the yardstick bid to the left of i^l, $z^{i^{l^l}}$ according to Definition 1 (or z^i alone if $z^{i^{l^l}} = z^0$). If $z^{i^{l^l}} = z^0$ it is clear that $\bar{x}^{i^l}$ increases when x^i increases (with the same amount). If $z^{i^{l^l}} \neq z^0$ we note that the facet spanned by $z^{i^{l^l}}$ and z^i becomes steeper as $z^{i^{l^l}}$ is fixed while z^i increases its cost component $x'^i > x^i$. Hence, $\bar{x}^{i^l}$ increases. Now, assume that i has a neighbor yardstick bid to the right $\bar{z}^{i^r}$ and let $z^{i^{r^r}}$ be its neighbor yardstick bid to the right (possibly z^P). The yardstick bid z^{i^r} is determined by z^i and $z^{i^{r^r}}$. Since the facet spanned by z^i and $z^{i^{r^r}}$ becomes flatter as i increases his cost component while $z^{i^{r^r}}$ remains fixed, we get that $\bar{x}^{i^r}$ increases.

Now, assume that the Principal's selection of preferred yardstick bid ($\bar{z}^{i*}$) is unchanged by i's increased bid. Then, since the yardstick bid of both neighbors increase if i increases his bid, all agents (except i) will get non-decreasing scores S_k^j while the scores of i, $S_k^i, k = 1, 2$ are non-increasing according to Definition 2. Thus, i's chance of winning the auction weakly decreases.

Also, note that agent i's potential compensation b^i decreases as a result of the manipulated scoring functions. Indeed, any potential scoring function to the left of i become flatter and any potential scoring functions to the right of i become stepper. Now a flatter scoring function to the left of i will cause a lower compensation if winning. This follows straightforward from the fact that the scoring function is spanned by $\bar{z}^{i^l}$ which increases by the manipulation and $\bar{z}^i$ which is unchanged by i's manipulation. Now, if i win with respect to this scoring function, the second highest scoring bid is to the left of i, which in turn result in a lower compensation to i. Likewise a steeper scoring function to the right will cause a lower compensation if winning. This also follows straightforward from the fact that the scoring function is spanned by $\bar{z}^{i^r}$ which increases by the manipulation and $\bar{z}^i$ which is unchanged by i's manipulation. Now, if i win with respect to this scoring function, the second highest scoring bid is to the right of i, which in turn results in a lower compensation to i.

Consequently, i has no incentives to bid above true cost. □

The next observation relates to the fact that even if the principal do change the preferred yardstick bid as a consequence of agent i's attempt to manipulate by increasing his cost bid, then this will only "rarely" be beneficial for i.

Proposition 5. *If the Principal's selection of preferred yardstick bid ($\bar{z}^{i*}$) is changed by agent i's increased cost bid, i has almost no incentives to bid above true cost.*

Proof. Now, contrary to proposition 4 the Principal's selection of preferred yardstick bid ($\bar{z}^{i*}$) is changed by i's increased bid. Then first we notice that the originally preferred yardstick bid must be a neighbor yardstick bid of $\bar{z}^i$ since only these bids are influenced. Say the originally preferred bid is $\bar{z}^{i^l}$. Given that the Principal uses linear scoring functions, the cone with vertex $\bar{z}^{i^l}$ spanned by $s^{i^{l^l}}$ and s^i (the scoring function of the left and right neighbor bids of $\bar{z}^{i^l}$ respectively) are relevant candidates for such a function.

By increasing x^i, agent i only influence the left and right neighbor yardstick bids. Therefore, if i^* is not a neighbor of i it has no influence on the Principal's selection. Let i^* be a neighbor of i or i itself. Clearly, if $i^* = i$ the Principal's selection remains the same as both neighbor yardstick bids increase. If i^* is either i^l or i^r the Principal selects a new preferred yardstick bid since both $\bar{z}^{i^l}$ and $\bar{z}^{i^r}$ increase.

It is clear that if the Principal change preferred yardstick bid to that of either the left neighbor of i^l or the right neighbor of i^r agent i will be worse off.

If i's increased cost bid (x'^i) causes the Principal to change its preferred yardstick bid from $\bar{z}^{i^r}$ to $\bar{z}^i$, agent i will be worse off (given that $x'^i \leq \bar{x}^i$). This follows from the same reasoning as used in the proof of Proposition 4. The steeper scoring function causes both i chance of winning the auction and the compensation if winning to decrease.

Now consider the situation where i's increased cost bid (x'^i) causes the Principal to change its preferred yardstick bid from $\bar{z}^{i^l}$ to $\bar{z}^i$ (given that $x'^i \leq \bar{x}^i$). Note that this exclude the otherwise flatter scoring function spanned by i^l and i^l's neighbor yardstick bid to the left. Hereby, i increases the chance of winning. However, By the same reasoning as used in the proof of Proposition 4, the steeper scoring function causes i compensation if winning to decrease. Although the possibilities are limited, above the increase that makes the Principal to change its preferred yardstick and below i's yardstick price, there may exist a situation where the right increase makes i better off.

Consequently, i has almost no incentives to bid above true cost. □

It follows from proposition 4 and 5 that bidding above true cost always results in a lower compensation if winning and in almost all situations, also a lower chance of winning. However, by proposition 5 we have that there exist situations where it pays for i to bid above true cost in order to win the auction. We will, however, argue that the room left for i to manipulate by bidding above true cost is almost negligible in practice. We have that manipulation by increasing always involves cost which may only be counteracted if the following two criteria are fulfilled. Criteria 1: it is only optimal for bidder i to bid above true cost if he is joint winner with the neighbor bidder to the left. Criteria 2: if criteria 1 is fulfilled, it is only optimal for bidder i to bid above true cost if it makes the Principal select i instead of the neighbor bidder to the left as the most preferred yardstick bid, otherwise the manipulation result in a loss as follows by the proof of proposition 4. If both criteria 1 and 2 is fulfilled, the manipulation involve a tradeoff between the chance of winning and the compensation if winning.

Proposition 6. *Bidders may have incentive to bid below true cost.*

Bidding below true cost causes the neighbor yardstick prices to decrease and consequently making the neighbor yardstick bids relatively more preferred to the Principal. Though, if i lowering the cost bid does not causes the Principal to change its preferred yardstick bid, the chance of i winning the auction and the compensation if winning increases (to a given point as illustrated in the example below). This follows by a similar set of arguments as the ones used in the proof of proposition 4. On the other hand, if i lowering the cost bid causes the Principal to change its preferred yardstick bid, we have the same effect for the most part. However, there may exist situations where it is not optimal to bid below true cost. This follows by a set of similar arguments as the ones used in the proof of proposition 5.

We will provide a proof of proposition 6 by an example. Consider the example already used to illustrate the auction and the following two situations where B decreases his otherwise truthful bid z^B to either $z^{B'}$ or $z^{B''}$ as illustrated in Figure 7. Bidding $z^{B'}$ illustrates a situation where B gain by bidding below true cost as oppose to bidding $z^{B''}$ where B losses by excluding the yardstick bid associated with A.

Consider the first situation where B's manipulated price bid ($z^B \rightarrow z^{B'}$) is above bidder A's price bid, i.e. A is assigned a yardstick bid. In this situation B's manipulation causes both neighbor yardstick bids downwards, which in turn results in new scoring functions. The new scoring functions both increases the chance of B winning as well as a higher compensation to B if winning. In the example, B becomes the sole winner in both before and after the manipulation. Therefore, the most profitable of the two second scores is chosen, which provides B with a higher compensation as a result of the steeper scoring function spanned by the lower $\bar{z}^{A'}$ and the unchanged $\bar{z}^B$.

Now, consider the second situation where B's manipulated price bid ($z^B \rightarrow z^{B''}$) is below bidder A's price bid, i.e. A is not assigned a yardstick bid. In this situation the effect of B's manipulation is divided into two: i) bidders to the right of B become relatively less competitive by the same logic as in the first situation and ii) bidders to the left of B become relatively more competitive by the new scoring function which results by the removing of the yardstick bid associated with bidder A. The new scoring functions lower the chance of B winning and results in a lower compensation to B if winning. In the example, B still becomes the sole winner both before and after the manipulation. Therefore, the most profitable of the two second scores is chosen, which actually provides B with a higher compensation as a result of the flatter scoring function spanned by the lower $\bar{z}^{C''}$ and the unchanged $\bar{z}^B$. However, B is worse off relative to the situation with the smaller manipulation ($z^{B'}$).

Although, bidding below true cost involves a risk of insufficient compensation (a risk that increases if more bidders follow the same bidding strategy), it may be optimal to bid slightly below true cost.

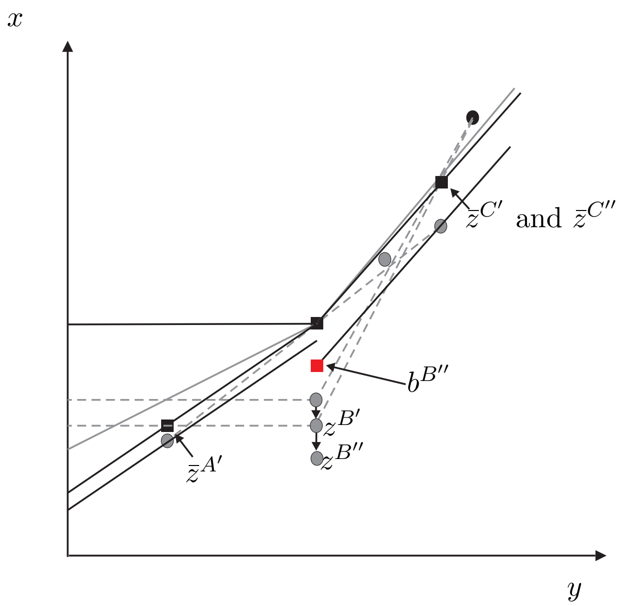

Figure 7: Optimal to bid sligthly below true cost

Observation 7. *The incentives to deviate from truthful bidding decreases with the number of participants.*

To see this, note that as the number of efficient bids increases the representation of the underlying cost structure improves. Therefore the chance that deviation from truth-telling will result in negative profit increases as the number of participants increases.

Since we have that potential manipulation in the suggested auction is almost limited to bidding below true cost, we also have that with few participants (and a weak representation of the underlying cost structure) it is more optimal to bid slightly below true cost and hereby lower the neighbor bidders' yardstick prices. Since this goes for all, we have that with few participants the bidders may bid slightly below true cost and enhance the otherwise smaller competition in a thin market.

Therefore with few participants the incentives to bid below the reservation price is relatively higher as oppose to a situation with many participants and with many participants the competition is enhanced by a better representation of the underlying cost structure. Hereby the auction enhance the competition in situation independently of the number of participants.

As a final remark, notice that since the auction provides incentive for the bidders to bid slightly below true cost, the auction is not individual rational. However, the compensation will always cover the costs if the bidder does not bid below true cost. To see this, note that it requires a positive score to win the auction and that a positive score is associated with a compensation weakly higher than the submitted cost-bid.

4.1 Example

To further illustrate the suggested auction we provide a numeric example below.

Consider a procurement situation where a Principal wants to buy "wired ethernet service" and let the only important attribute be the price (cost) and the internet speed measured in Mbit/s (speed). The Principal clearly prefers high speed and low cost, we will, however, assume that the Principal is uncertain about his actual weighting of speed and cost. Also, to keep it simple, we will assume that all other issues regarding the purchase of the internet service is fixed for all participants (sellers). Figure 8 provides the bids from the 4 participants. Figure 8 also provide the yardstick bid for bidder B. As illustrated, the yardstick price $\bar{x}^B = 484,4$ is given as the convex combination of the two bids $z^A = (240, 4)$ and $z^D = (680, 14, 8)$.

Figure 9 illustrates the situation where the Principal has selected $\bar{z}^B = (484, 4, 10)$ as the most preferred yardstick bid among the three feasible yardstick bids. $\bar{z}^B$ functions as vertex for the cone of potential linear scoring functions and the applied scoring functions s_1 and s_2 is pictured in Figure 9. The two scoring functions are given as:

$s_1 = a_1 + b_2 y^i = 277,3 + 20,7y^i$
$s_2 = a_2 + b_2 y^i = -157,7 + 64,2y^i$

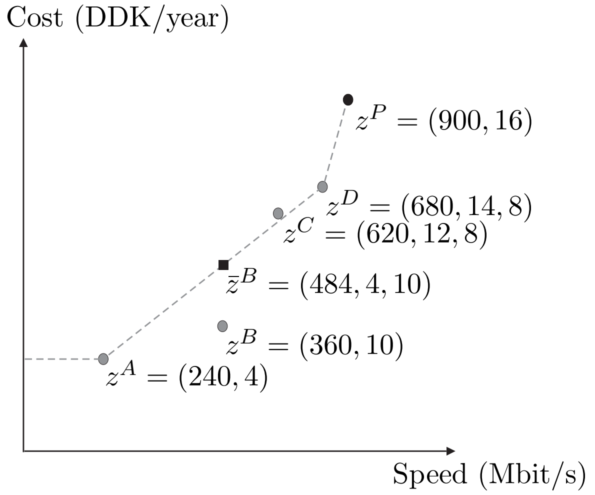

Figure 8: Initial bidding and yardstick prices

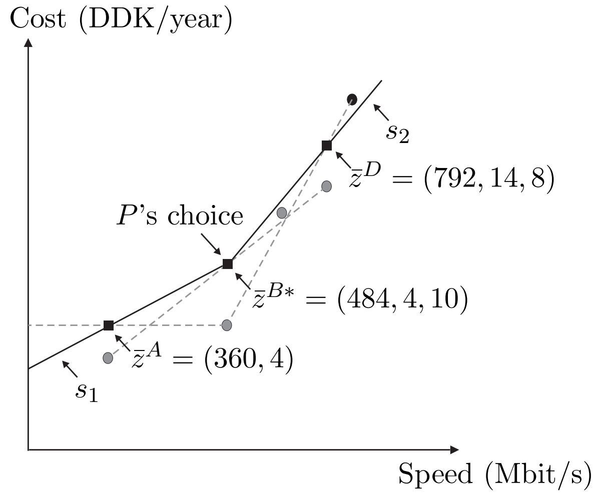

Figure 9: Yardstick prices and the scoring functions

Now the original bids are scored with respect to the two scoring functions. As an example, A is given the score $S_1^A = s_1^A - x^A = (277,3 + 20,7 \cdot 4) - 240 = 120,1$. Table 1 provides the different scores for the 4 participants and the two scoring functions. From the scores in Table 1 it is clear that B is the sole winner by having the highest score with respect to both scoring functions. A and D holds the second highest score for the two scoring functions s_1 and s_2 respectively.

Figure 10 illustrates the scoring and how the winner is selected and how the compensation is determined. Since B is the sole winner there are two second highest scores that define the following two different compensations:

$b_1^{(1),A} = s_1^i - S_1^{(2)} = (277,3 + 20,7 \cdot 10) - 120 = 364,3$
$b_2^{(1),A} = s_2^i - S_2^{(2)} = (-157,7 + 64,2 \cdot 10) - 112,5 = 371,8$ (applied)

Table 1: The resulting scores

Bidders	S_1^i	S_2^i	$S_k^{(1)}$ and $S_k^{(2)}$
A	120,1	-140,9	$S_1^{(2)}$
B	124,3	124,3	$S_1^{(1)}$ and $S_2^{(1)}$
C	-77,7	44,1	
D	-96,3	112,5	$S_2^{(2)}$

As the auction rules dictates, the most preferred of the two from the winner's perspective is applied.

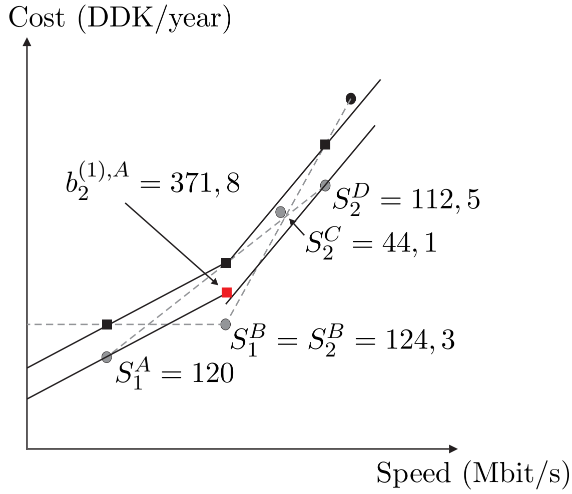

Figure 10: The scoring, selection and compensation

5 Extensions and Discussion

In this section a brief discussion of future extensions of the suggested auction is discussed.

The suggested auction is designed to handle a situation where only a single value of the attribute can be produced for each agent, i.e., $Y^i = \{y^i\}$ for all $i \in N$. Although there might exist situations where specific investments restricts the bidders to a single value of y, this is by no means true in general.

One approach is to restrict the bidders to only submit a single bid. This however, imposes a problem to bidder i of selecting the most optimal bid with respect to the competition facilitated by the auction. Depending on the situation, it might be the case that a bidder ex post would have preferred to submit a different bid. Apart from this, the properties of the auction will be the same.

Another approach is the one applied in the paper Bogetoft and Nielsen (2008), where the bidders are allowed to submit a corresponding set of bids, i.e. agent i will submit $(x^i(y^i), y^i)$ for all or some of the possibilities $y^i \in Y^i$. To address strategic manipulation of the yardstick prices, the well-defined scoring function is used to screen the submitted bids before they are being used in the computation of the yardstick prices. However, this is not a feasible approach in our setting since no single scoring function exist a priori, and since the applied scoring functions are functions of the estimated yardstick prices. Therefore, allowing each bidder to submit multiple bids requires a different approach. This approach shall handle the fact that a bidder may have more most preferred yardstick bids relative to the other bidders and that the same bidder may have more candidates to become the left or/and right neighbor yardstick bids.

Another important extension is to allow y to be multidimensional. This cause no problems in computing the applied yardstick prices. However, the interpretation of the Principal's choice of a single preferred yardstick bid as well as the deduced piecewise linear scoring functions (which would be hyperplanes and not lines) are different. Our expectation is that the extension from 2 to 3 dimensions is demanding.

Another important related research is that of ensuring a secure and cost efficient implementation. Keeping the submitted bids confidential is crucial for the mechanism and this challenge a cost efficient implementation. However, ongoing research in "secure multiparty computation" aim at implementing encrypted linear programming which would be sufficient to compute the applied yardstick bids, see e.g. Nielsen and Toft (2007) for an introduction. Also in practical procurement, having the bids secured throughout the auction process, allows for a simple and cost efficient information sharing required to involve internal or external personal in the important evaluation of the non-price attributes.

6 Conclusions

The suggested auction mechanism simplifies the balancing of costs and benefits by postponing the Principal's articulation of preferences to after reviewing the actual offers containing competitive yardstick prices. Hereby the auction provides some of the same flexibility as a traditional negotiation process without lowering the transparent competition offered by the auction rules. By selecting a most preferred yardstick bid, a set of second score auctions are performed, which results in final prices that are more competitive than the initial yardstick prices and reflects the preferences deduced by the Principal's initial choice.

The auction provides almost ideal incentives for bidders to bid the true cost or less. The incentives is created by the use of the two revelation mechanisms; the initial yardstick prices and final second scoring. The incentives to bid below true cost is largest in thin markets, which in turn enhance the competition in the most critical market situations.

Future research will focus on extending the auction to handle more realistic market situations.

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